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$$\frac{x_1^2}{a^2} + \frac{y_1^2}{b^2} + \frac{z_1^2}{c^2} = 1 = \frac{x_2^2}{a^2} + \dots = \frac{x_3^2}{a^2} + \dots \quad (7),$$

$$\text{and } x_1y_1 + x_2y_2 + x_3y_3 = y_1z_1 + \text{etc.}, = z_1x_1 + \text{etc.}, = 0 \dots \quad (8),$$

$$l/p = \frac{x_1 + y_1 + z_1}{a^2}, \quad m/p = \text{etc.}, \quad n/p = \text{etc.}, \dots \quad (9).$$

These must be put into (1).

Also solved by *G. B. M. ZERR, J. W. YOUNG, LON C. WALKER, J. SCHEFFER, and GEORGE LILLEY.*

128. Proposed by *W. H. CARTER*, Vice President and Professor of Mathematics, Centenary College, Jackson, La.

Given $F = \Delta^{n-1} \div (n-1)! \cdot \Delta_1 \cdot \Delta_2 \dots \Delta_n$, where Δ = the determinant $(a_1 b_2 c_3 \dots k_n)$ and $\Delta_1 \Delta_2 \dots \Delta_n$ are the minors of the elements of the n th column; and $a_m, b_m, c_m \dots$ etc., ($m=1, 2, 3 \dots n$) are the coefficients of n given equations containing $n-1$ variables. Show (1) that $n=3$, F = the area of a triangle, and (2) if $n=4$, F = the volume of the tetrahedron.

Solution by *J. W. YOUNG*, Student in Ohio State University, Columbus, O.

1. Let $n=3$. The points of intersection of the three lines represented by the given equations, are

$$x_1 = -\frac{A_1}{C_1}; \quad x_2 = -\frac{A_2}{C_2}; \quad x_3 = -\frac{A_3}{C_3};$$

$$y_1 = -\frac{B_1}{C_1}; \quad y_2 = -\frac{B_2}{C_2}; \quad y_3 = -\frac{B_3}{C_3};$$

where, by the usual notation, A_k equals the co-factor a_k , in the determinant Δ .

The area of the triangle formed by these points is

$$\text{Area} = \frac{1}{2} \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = \frac{1}{2} \begin{vmatrix} -\frac{A_1}{C_1} & -\frac{B_1}{C_1} & 1 \\ -\frac{A_2}{C_2} & -\frac{B_2}{C_2} & 1 \\ -\frac{A_3}{C_3} & -\frac{B_3}{C_3} & 1 \end{vmatrix} = \frac{1}{2} \begin{vmatrix} A_1 & B_1 & C_1 \\ A_2 & B_2 & C_2 \\ A_3 & B_3 & C_3 \end{vmatrix} \div C_1 C_2 C_3$$

and this, by a well-known theorem in determinants,

$$= \frac{1}{2} \Delta^2 \div C_1 C_2 C_3 = F.$$

2. Let $n=4$. The intersections of the four planes given by the equations are found precisely as above.

The volume of the tetrahedron found by the points is

$$\frac{1}{3!} \begin{vmatrix} x_1, & y_1, & z_1, & 1 \\ x_2, & y_2, & z_2, & 1 \\ x_3, & y_3, & z_3, & 1 \\ x_4, & y_4, & z_4, & 1 \end{vmatrix}$$

or substituting the values of x_1, y_1, z_1 , etc., we have

$$\begin{aligned} \text{Volume} &= \frac{1}{6} \begin{vmatrix} A_1, & B_1, & C_1, & D_1 \\ A_2, & B_2, & C_2, & D_2 \\ A_3, & B_3, & C_3, & D_3 \\ A_4, & B_4, & C_4, & D_4 \end{vmatrix} \div D_1 D_2 D_3 D_4 \\ &= \frac{1}{6} \Delta^3 \div \Delta_1 \Delta_2 \Delta_3 \Delta_4 = F. \end{aligned}$$

Also solved by *G. B. M. ZERR, WALTER H. DRANE*, and the *PROPCSER*.
Professor Carter asks: What does F represent when n is greater than 4?

CALCULUS.

97. Proposed by *ARTEMAS MARTIN, A.M., Ph.D., LL.D.*, United States Coast and Geodetic Survey Office, Washington, D. C.

An auger hole, radius r , is bored through a prolate spheroid; the axis of the auger passing through the center, perpendicular to the major axis. Find the volume removed.

Solution by *G. B. M. ZERR, A. M., Ph. D.*, Professor of Mathematics and Science, Chester High School, Chester, Pa.

Let $\frac{x^2}{a^2} + \frac{y^2 + z^2}{b^2} = 1$, be the equation to the prolate spheroid.

$x^2 + y^2 = r^2$, the equation to the cylinder.

$$\begin{aligned} \therefore V &= 8/a \int_0^r \int_0^{\sqrt{r^2 - x^2}} \sqrt{[b^2(a^2 - x^2) - a^2 y^2]} dx dy \\ &= 4/a \int_0^r \{ (r^2 - x^2) [a^2(b^2 - r^2) + (a^2 - b^2)x^2] \} dx \\ &\quad + \frac{4b^2}{a^2} \int_0^r (a^2 - x^2) \sin^{-1} \left[\frac{a}{b} \sqrt{\left(\frac{r^2 - x^2}{a^2 - x^2} \right)} \right] dx \\ &= 4/a \int_0^r \{ (r^2 - x^2) [a^2(b^2 - r^2) + (a^2 - b^2)x^2] \} dx \\ &\quad + \frac{4b^2(a^2 - r^2)}{3a} \int_0^r \frac{x^2 dx}{\sqrt{\{ (r^2 - x^2) [a^2(b^2 - r^2) + (a^2 - b^2)x^2] \}}} \\ &\quad + \frac{8ab^2(a^2 - r^2)}{3} \int_0^r \frac{x^2 dx}{(a^2 - x^2) \sqrt{\{ (r^2 - x^2) [a^2(b^2 - r^2) + (a^2 - b^2)x^2] \}}} \end{aligned}$$